

Solar neutrinos and the solar composition problem

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Standard solar models (SSM) are facing nowadays a new puzzle: the solar composition problem. New determinations of solar metal abundances lead SSM calculations to conflict with helioseismological measurements, showing discrepancies that extend from the convection zone to the solar core and can not be easily assigned to deficiencies in the modelling of the solar convection zone. We present updated solar neutrino fluxes and uncertainties for two SSM with high (old) and low (new) solar metallicity determinations. The uncertainties in iron and carbon abundances are the largest contribution to the uncertainties of the solar neutrino fluxes. The uncertainty on the $^{14}\text{N}(p,\gamma)^{15}\text{O}$ rate is the largest of the non-composition uncertainties to the CNO neutrino fluxes. We propose an independent method to help identify which SSM is the correct one. Present neutrino data can not distinguish the solar neutrino predictions of both models but ongoing measurements can help to solve the puzzle.

This paper is part of a series led by John Bahcall that spans more than 40 years [1]. The goals of this series have been to provide increasingly more precise theoretical calculations of the solar neutrino fluxes and detection rates and to make increasingly more comprehensive evaluations of the uncertainties in the predictions.

In the past, standard solar models (SSM) had been steadily showing a discrepancy with neutrino measurements, named the solar neutrino problem, that definitively led to the discovery of flavor change in the lepton sector. The significant difference of the charged current, electron scattering and neutral current measurements of the epochal Sudbury Neutrino Observatory (SNO) and Super-Kamiokande experiments have demonstrated the new phenomenon. As soon as the solar neutrino puzzle was solved, a new and different puzzle has emerged.

Recent refined determinations of the abundances of heavy elements on the solar surface have led to lower heavy-element abundances [2] than previously measured. Solar models that incorporate these lower abundances conflict dramatically with helioseismological measurements. Detailed studies of the physical inputs have been done to resolve this controversy; with particular focus on the region where more dramatically low metallicity solar models fail to explain helioseismological measurements, right below the solar convective envelope. But, recent analysis of precise low-degree acoustic oscillations of the Sun measured by the Birmingham Solar-Oscillations Network (BiSON) indicate that the discrepancies between solar models constructed with low metallicity and helioseismic observations extend to the solar core and thus cannot be attributed to deficiencies in the modelling of the solar convection zone [3].

A number of studies have been done to refine the estimated uncertainties and search for systematic effects that could account for the disagreement of helioseismological predictions of the low metallicity solar models. Today,

we lack a solution to this puzzle and extra handles are needed to identify the source of the discrepancy. In this letter, we propose a new method to discriminate whether the low heavy-element abundances should be accepted with the other solar inputs in the SSM or not.

Here we present the most updated solar model calculations with the two solar abundance determinations, high metallicity labelled as BPS08(GS) [4] and low metallicity labelled as BPS08(AGS) [2]. There are two steps forward that contribute to major improvements in the estimated precision of the neutrino fluxes: i) improved accuracy of the ^3He - ^4He cross section and ii) reduced systematic uncertainties in the determination of the surface composition of the Sun.

Previously, the uncertainty on the S-factor of the $^3\text{He}(\alpha,\gamma)^7\text{Be}$ reaction was due to an average discrepancy in results obtained by the detection of the delayed γ rays from ^7Be decay and the measurement of prompt γ emission. LUNA collaboration reported on a new high precision experiment using both techniques simultaneously at the lowest interaction energy ever reached in the lab. The S-factors from the two methods do not show any discrepancy within the experimental errors, obtaining $S(0)=0.567\pm0.018\pm0.004$ keV barn [5]. With this new determination, the uncertainty on the predicted ^8B (^7Be) neutrino flux due to S_{34} is reduced from 7.5% to 2.7% (8.0% to 2.8%). Best-estimates and $1-\sigma$ uncertainties of the other important astrophysical factors are summarized in Table 1 of Ref. [6], with minor modifications [7].

Previously, we preferred solar model calculations with conservative uncertainties in the element abundances, estimated by the difference between the high and low metal abundances, although it was recognized that estimated uncertainties based on each analysis independently were smaller [9]. New detailed studies of helioseismological data [3] have shown that low-degree helioseismology data clearly allow to discriminate between predictions for the

TABLE I: Predicted solar neutrino fluxes from solar models. The table presents the predicted fluxes, in units of $10^{10}(pp)$, $10^9(^7\text{Be})$, $10^8(pep, ^{13}\text{N}, ^{15}\text{O})$, $10^6(^8\text{B}, ^{17}\text{F})$, and $10^3(hep)$ $\text{cm}^{-2}\text{s}^{-1}$. Columns 2 and 3 show BPS08 for high and low metallicities; and column 4 the flux differences between the models.

Source	BPS08(GS)	BPS08(AGS)	Difference
<i>pp</i>	$5.97(1 \pm 0.006)$	$6.04(1 \pm 0.005)$	1.2%
<i>pep</i>	$1.41(1 \pm 0.011)$	$1.45(1 \pm 0.010)$	2.8%
<i>hep</i>	$7.90(1 \pm 0.15)$	$8.22(1 \pm 0.15)$	4.1%
^7Be	$5.07(1 \pm 0.06)$	$4.55(1 \pm 0.06)$	10%
^8B	$5.94((1 \pm 0.11)$	$4.72(1 \pm 0.11)$	21%
^{13}N	$2.88(1 \pm 0.15)$	$1.89(1 \pm^{+0.14}_{-0.13})$	34%
^{15}O	$2.15(1 \pm^{+0.17}_{-0.16})$	$1.34(1 \pm^{+0.16}_{-0.15})$	31%
^{17}F	$5.82(1 \pm^{+0.19}_{-0.17})$	$3.25(1 \pm^{+0.16}_{-0.15})$	44%
Cl	$8.46^{+0.87}_{-0.88}$	$6.86^{+0.69}_{-0.70}$	
Ga	$127.9^{+8.1}_{-8.2}$	$120.5^{+6.9}_{-7.1}$	

solar core structure from solar models with high and low metallicity. High metallicity models agree with helioseismological data but not with recent determinations of solar abundances. On the contrary, low metallicity models are inconsistent with helioseismological data but use the recent determinations of solar abundances. That is, high and low metallicity solar models really are two different groups of models, and conservative uncertainties defined as described above are, in fact, too conservative. We find that uncertainties have to be estimated for each class independently. In what follows, we adopt for each solar abundances set [2, 4] both the recommended values and uncertainties for each chemical element.

Table I presents, in the second and third columns, labelled BPS08(GS) and BPS08(AGS), our best solar model calculations for the neutrino fluxes. Both models were calculated with the new input data for nuclear physics [5, 7]. BPS08(GS) was computed with the high (old) metallicity solar abundances determinations [4] and BPS08(AGS) with the most recent analyses of solar abundances [2] that lead to conflicts with helioseismological measurements.

Table I also shows the overall uncertainties in the neutrino fluxes. Uncertainties from different sources have been calculated using power-law expansions with coefficients given in Table II and added quadratically. Uncertainties due to errors in the solar composition (metals) have been determined for each relevant metal separately [9] and abundance uncertainties are taken from the respective solar composition compilations [8]. This is a more consistent way for estimating fluxes uncertainties and leads to, incidentally, reduced overall uncertainties. This result is particularly relevant in the context of CNO fluxes and the solar abundance problem. Solar composition dominates the uncertainty in the CNO fluxes (13% for ^{13}N , 12% for ^{15}O and 17% for ^{17}F) and it is not dom-

inant for all other fluxes. Among the non-composition sources, the most important contribution to the total error in the CNO fluxes is due to the $S_{1,14}$ uncertainty. $S_{1,14}$ contributes with 6% (8%), while diffusion and opacity contribute with 4% and 5% (5% and 6%) to the ^{13}N (^{15}O) neutrino flux. For the ^7Be flux, opacity (3.2%) contributes most to the total error, followed by S_{34} and S_{33} (2.8% and 2.5%); while for ^8B , opacity (6.8%), diffusion (4.2%) and S_{17} (3.8%) dominate the uncertainties.

A careful look to the logarithmic partial derivatives with respect to the individual solar abundances and the uncertainties of the individual elements shows that the major contribution to the uncertainties in the ^{13}N and ^{15}O neutrino fluxes largely comes from the uncertainty in the carbon abundance. Therefore, a reduction of the carbon abundance uncertainty directly translates into a substantial improvement of the CNO neutrino flux uncertainties. On the other hand, ^8B and ^7Be fluxes are much more sensitive to the iron abundance (due to its influence on the core opacity) and therefore a reduction of the iron abundance uncertainty directly translates into a substantial improvement of the ^8B and ^7Be neutrino flux uncertainties.

Finally, in Table II we also included power-law coefficients for the dependence on two important helioseismic quantities on solar model input parameters, which show the relevant inputs that contribute to their uncertainties. SSM predictions for these quantities are $R_{CZ} = 0.713(0.728) \pm 0.004$ and $Y_S = 0.243(0.229) \pm 0.035$ for the high (low) metallicity models; while helioseismologically determined values are $R_{CZ} = 0.713 \pm 0.001$ and $Y_S = 0.2485 \pm 0.0034$ [10].

Opacity uncertainties have been conservatively computed by defining the $1\text{-}\sigma$ uncertainty as given by the flux differences between two SSMs that have been computed one with standard opacities and the other with opacities in the solar interior increased by 2.5%. Such a difference in the solar core opacities is representative of the maximum difference between the two most up-to-date radiative opacity calculations, OPAL and OP [11].

Based on the results presented above, we make two recommendations for future work to improve the uncertainties in the SSM neutrino fluxes. First, the uncertainty in the low energy extrapolation of the rate of the $^{14}\text{N}(p,\gamma)^{15}\text{O}$ reaction should be reduced to below 5% (almost a factor 2 improvement) in order that the uncertainty due to astrophysical cross sections does not dominate the non composition uncertainties in any of the neutrino fluxes. Second, the uncertainty in the surface element abundances iron and carbon should be reduced to ± 0.02 dex (factor 2 to 3 improvement) to significantly improve the radiative opacity and the CNO composition uncertainties. The requirements are tough, but the outcome is rewarding.

Table I shows that the central values of the neutrino fluxes in the two models BPS08(GS) and BPS08(AGS)

TABLE II: Logarithmic partial derivatives $\alpha(i, j)$ of neutrino fluxes and helioseismic quantities with respect to nuclear cross sections, solar luminosity and age, element diffusion rate, radiative opacity and solar abundances.

Source	S ₁₁	S ₃₃	S ₃₄	S ₁₇	S _{hep}	S _{1,14}	S _{7Be,e}	L _⊙	Age	Diff	Opac	C	N	O	Ne	Mg	Si	S	Ar	Fe
<i>pp</i>	0.090	0.029	-0.059	0.000	0.000	-0.004	0.000	0.808	-0.067	-0.011	-0.099	-0.005	-0.001	-0.005	-0.004	-0.004	-0.009	-0.006	-0.001	-0.016
<i>pep</i>	-0.236	0.043	-0.086	0.000	0.000	-0.007	0.000	1.041	0.017	-0.016	-0.300	-0.009	-0.002	-0.006	-0.003	-0.002	-0.012	-0.014	-0.003	-0.054
<i>hep</i>	-0.112	-0.459	-0.072	0.000	1.000	-0.004	0.000	0.174	-0.118	-0.037	-0.398	-0.007	-0.002	-0.020	-0.014	-0.017	-0.036	-0.028	-0.005	-0.064
⁷ Be	-1.07	-0.441	0.878	0.000	0.000	-0.001	1.000	3.558	0.786	0.136	1.267	0.004	0.002	0.053	0.044	0.057	0.116	0.083	0.014	0.217
⁸ B	-2.73	-0.427	0.846	1.000	0.000	0.005	0.000	7.130	1.380	0.280	2.702	0.025	0.007	0.111	0.083	0.106	0.211	0.151	0.027	0.510
¹³ N	-2.09	0.025	-0.053	0.000	0.000	0.711	0.000	4.400	0.855	0.340	1.433	0.861	0.148	0.047	0.035	0.051	0.109	0.083	0.015	0.262
¹⁵ O	-2.95	0.018	-0.041	0.000	0.000	1.000	0.000	6.005	1.338	0.394	2.060	0.810	0.207	0.075	0.055	0.076	0.158	0.117	0.021	0.386
¹⁷ F	-3.14	0.015	-0.037	0.000	0.000	0.005	0.000	6.510	1.451	0.417	2.270	0.024	0.005	1.083	0.061	0.084	0.174	0.128	0.023	0.428
<i>R_{CZ}</i>	-0.061	0.002	-0.003	0.000	0.000	0.000	0.000	-0.016	-0.081	-0.018	-0.012	-0.006	-0.005	-0.028	-0.012	-0.005	0.002	0.004	0.001	-0.009
<i>Y_S</i>	0.134	-0.005	0.009	0.000	0.000	0.001	0.000	0.373	-0.110	-0.073	0.646	-0.009	-0.001	0.023	0.033	0.037	0.070	0.048	0.009	0.089

differ by 2-3 times the theoretical error expected within one model. This feature must be explored to estimate by how much solar neutrino experiments can contribute to solve the solar composition puzzle. Two of the neutrino fluxes (⁸B and ⁷Be neutrinos) have been measured with good precision, and this can lead to preliminary tests of the solar composition. SNO collaboration [12, 13, 14] has determined the ⁸B neutrino flux with good precision by the measurement of the charged current and the neutral current detection of solar neutrinos in three phases of the experiment: I) heavy water target, II) heavy water + salt target and III) heavy water + ³He targets. All three phases lead to a very precise measurement of the ⁸B flux ($\sim 6\%$). Moreover, Super-Kamiokande electron scattering data, when combined with the other solar neutrino data and the assumption of neutrino oscillations, contributes to further reduce the uncertainty in the ⁸B flux. Since May 2007, the Borexino detector is collecting data on the electron scattering of ⁷Be neutrinos. The collaboration presented the latest results on ⁷Be solar neutrinos ($\sim 10\%$ precision) based on 192 live days of data taking [15]. With more statistics and better calibrations, the collaboration anticipates a final precision at the 5% level.

In order to quantitatively determine how well the two solar models fit the neutrino data we proceed in two steps: a) use all of the available solar and reactor neutrino data to determine the current constraints on neutrino oscillation parameters and solar neutrino fluxes with the method described in Ref. [16], b) use the solar neutrino fluxes inferred from data to test SSM fluxes. We discuss now which data have been included in the analysis and refer to Ref. [16] for further details. Solar neutrino data includes the average rate measured by the radiochemical experiments, chlorine [17] and gallium [18], the SuperKamiokande I zenith spectrum [19], the SuperKamiokande II day-night spectrum [20], the SNO pure D₂O phase day-night spectrum [12], the SNO salt phase day-night spectrum [13], the electron scat-

tering, charged current and neutral current rates measured in phase 3 [14] and the Borexino ⁷Be and ⁸B rates [15]. Non solar neutrino data include the antineutrino data from KamLAND [21] and the marginalized $\chi^2(\theta_{13})$ function derived from the analysis of atmospheric+K2K+MINOS+CHOOZ data [22]. The free parameters in the analysis are the neutrino parameters Δm_{21}^2 , θ_{12} and θ_{13} and the reduced neutrino fluxes f_B , f_{Be} , f_{pp} , f_N and f_O (f_i is the i -th neutrino flux normalized to the BPS08(GS) prediction) subject to the luminosity constraint, i.e., the total luminosity produced by nuclear reactions (the p-p chains and the CNO cycle) equals the observed solar luminosity. We find that:

$$\begin{aligned}
 \Delta m_{21}^2 &= (7.7 \pm 0.2) \times 10^{-5} \text{eV}^2 \\
 \tan^2 \theta_{12} &= 0.46_{-0.05}^{+0.04} ; \sin^2 \theta_{13} = 0.014_{-0.009}^{+0.011} \\
 f_B &= 0.91 \pm 0.03 ; f_{Be} = 1.02 \pm 0.10 \\
 f_{pp} &= 1.00_{-0.02}^{+0.01} ; L_{CNO} = 0.0_{-0.0}^{+2.9\%}
 \end{aligned}$$

where L_{CNO} is the fraction of solar luminosity due to the CNO nuclear fusion reactions. The oscillation parameters, central values and errors, are in excellent agreement with neutrino analysis where the solar neutrino fluxes are not treated as free variables [22, 23]. Compared to previous analysis [16], the major improvement is in the determination of the ⁷Be flux. The Borexino measurements lead to a factor five improvement of the uncertainty in this flux. The upper bound on the CNO luminosity is not significantly different [24] because the improvement due to the ⁷Be determination leading to the more constrained ¹³N and ¹⁵O fluxes, compensates the lower CNO fluxes predicted by the solar model calculations using the most accurate cross section of the ¹⁴N(p, γ)¹⁵O reaction [7] used in BPS08. The results of this analysis allow us to experimentally determine the ratio of the termination chains in the p-p burning: $R = \langle ^3\text{He} + ^4\text{He} \rangle / \langle ^3\text{He} + ^3\text{He} \rangle = 0.19 \pm 0.02$.

The global analysis of neutrino data used above was discussed in Refs. [16, 24] with the goal to derive neu-

trino oscillation parameters and solar neutrinos fluxes from neutrino data independently of the solar model. We can now move to the second step in the analysis and test SSM predictions without dealing with solar neutrino data, which is encoded in the results presented above. In order to statistically test the hypothesis that there is no difference between a given solar model and the data, we define the χ^2 function

$$\chi^2 = \sum_{ij} (f_i^{th} - f_i) \sigma_{ij}^{-2} (f_j^{th} - f_j)$$

$$\sigma_{ij}^2 = \sigma_{exp,i} \sigma_{exp,j} \rho_{ij}^{exp} + \sigma_{th,i} \sigma_{th,j} \rho_{ij}^{th}$$

where $\sigma_{exp,i}$ and $\sigma_{th,i}$ are the experimental and theoretical errors of the neutrino flux f_i and ρ_{ij}^{exp} and ρ_{ij}^{th} are the experimental and theoretical correlations of the fluxes i and j . Given the large uncertainties in some of the experimentally determined fluxes, we only include in the analysis the most precise ones (^8B and ^7Be fluxes), which have negligible experimental correlations. Theoretical correlation coefficients for neutrino fluxes can be found in Ref. [8]. The analysis leads to:

$$\chi_{BPS08(GS)}^2 = 0.9 \text{ (63\%)} \text{ and } \chi_{BPS08(AGS)}^2 = 1.5 \text{ (47\%)},$$

where the goodness of fit is shown in parenthesis and is used as a diagnostic for the fit of the model to the data. Neutrino data are not precise enough to reject the null hypothesis for any of the models, BPS08(GS) and BPS08(AGS). Adding the pp flux in the analysis does not improve the statistical power, χ^2 changes to 0.95 and 1.6 respectively. These results show that the theoretical and statistical errors are still too large to reject any of the two models. The analysis proposed here will guide us on how improvements can contribute to solve the composition problem. Eventually, the rejection of the null hypothesis would prompt us to use a likelihood ratio testing for a given parameter(s) connecting the models. More Super-Kamiokande data and the full analysis of the SNO data, with lower energy threshold, will sharpen the accuracy of the ^8B neutrino flux. Borexino ongoing measurements anticipate further improvements of the statistical and systematic errors which may help to discriminate models. To illustrate what might be achieved, if we assume that future neutrino data will lead to $f_B = 1.0 \pm 0.02$ and $f_{Be} = 1.00 \pm 0.05$, and theoretical errors on the ^8B and ^7Be fluxes are reduced from ± 0.11 and ± 0.06 to ± 0.08 and ± 0.04 respectively, we obtain $\chi_{BPS08(GS)}^2 = 0$ (by construction) and $\chi_{BPS08(AGS)}^2 = 6.2$ (4.5%), what will hint towards the relevance of doing an analysis of solar models as a function of the iron composition to all neutrino data. Moreover, running and planned low energy experiments have a realistic chance to measure CNO neutrinos and further distinguish the generic predictions of both models, given that predictions of BPS08(GS) and BPS08(AGS) for these fluxes differ about 30%. This kind

of study, however, is beyond the scope of this paper. The hypothesis testing analysis with future data will show whether this effort is valuable to discriminate between the high and low composition SSM.

In summary, standard solar models are facing the solar composition problem, i.e., new composition determinations, when included in solar models, conflict with helioseismological measurements. We are agnostic whether the new composition, the solar modelling, other inputs or a combination of them is the source of the conflict. We have proposed an independent method to identify which solar model is the correct one by using solar neutrino data. A method of analysis was proposed and preliminary results have been derived. Ongoing solar neutrino measurements and forecast neutrino flux measurements will help in the search of the solution to this puzzle.

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